A Blind Antenna Selection Scheme for Single-Cell Uplink Massive MIMO

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Abstract

We approach the problem of antenna selection in single-cell uplink massive MIMO using two different techniques. The first one consists in solving a convex relaxation of the problem using standard convex optimization tools. The second technique solves the problem using a greedy approach. The main advantages of the greedy approach lies in its wider scope, in that, unlike the first approach, it can be applied irrespective of the considered performance criterion. In the case where we don't have perfect channel knowledge, We extend both approaches to perform blind antenna selection that is only based on the channel statistics.

1. System Description

1.1 Downlink Model

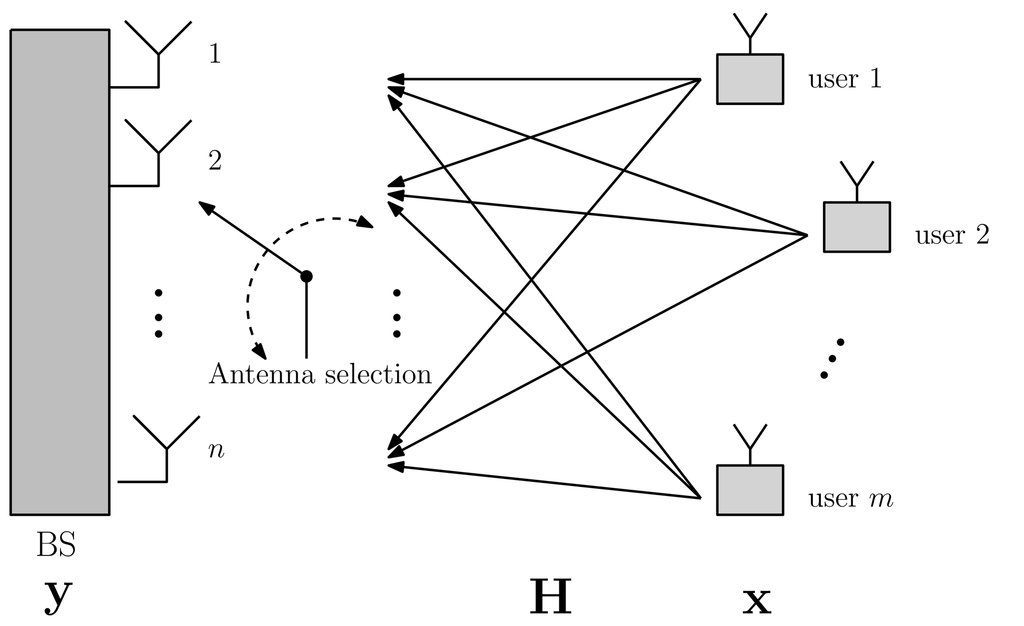

Figure 1: *System model of an uplink MU-MIMO system composed of a BS equipped with n antennas and serving m single-antenna users.*

The received signal vector at the BS is given by

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{e}. \tag{1}$$

The random channel $\mathbf{H}$ exhibits the one-sided Kronecker model given by

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}}\mathbf{W}, \tag{2}$$

where $\mathbf{W}$ is a matrix with $i.i.d$, $\mathcal{CN}(0,1)$ entries.

At the output of the zero-forcing receiver (ZF), the estimated signal is given by

$$\hat{\mathbf{x}} = \frac{1}{\sqrt{\rho}}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{y} \tag{3}$$

$$\begin{aligned}\mathrm{MSE} &= \mathbb{E}\left[\|\hat{\mathbf{x}} - \mathbf{x}\|^2\right] \\ &= \frac{1}{\rho}\mathrm{tr}\left[\left(\mathbf{W}^H\mathbf{R}\mathbf{W}\right)^{-1}\right].\end{aligned} \tag{4}$$

2. Antenna Selection

2.1 CSI-aware Antenna Selection

Let

$$\mathrm{MSE}(\mathbf{s}) = \mathrm{tr}\left[\left(\mathbf{H}^H \mathrm{diag}(s)\,\mathbf{H}\right)^{-1}\right]. \tag{5}$$

Then, the selection problem is formulated as follows

$$\begin{aligned}&\underset{\mathbf{s}}{\text{minimize}} \quad \mathrm{MSE}(\mathbf{s}) \\ &\text{s.t.} \qquad \mathbf{1}^T\mathbf{s} = k \\ &\qquad\qquad s_i \in \{0,1\},\ i = 1,\cdots,n.\end{aligned} \tag{6}$$

2.1.1 Antenna Selection via Convex Optimization

It is mainly based in relaxing the boolean constraints in (6)

$$\begin{aligned}&\mathbf{s}_0 = \underset{\mathbf{s}}{\text{argmin}} \quad \mathrm{MSE}(\mathbf{s}) \\ &\text{s.t.} \qquad \mathbf{1}^T\mathbf{s} = k \\ &\qquad\qquad 0 \le s_i \le 1, i = 1,\cdots,n.\end{aligned} \tag{7}$$

2.1.2 Greedy Approach-based Antenna Selection

0: **Initialize** $\mathcal{S} = randsample(n,k)$
0: **Compute** $\mathrm{MSE}^* = f(\mathcal{H},\mathcal{S})$
1: **for** $i = 1$ **to** $\#\ iterations$ **do**
1: $\quad \overline{\mathcal{S}} = \{1,\cdots,n\}\backslash\mathcal{S}$
1: $\quad j \leftarrow 1$
2: $\quad$ **while** $j \le n - k$ **do**
2: $\quad\quad p \leftarrow \overline{\mathcal{S}}[j]$
2: $\quad\quad \mathcal{I} \leftarrow \mathcal{S}$
2: $\quad\quad$ **table** $\leftarrow$ **zeros** $(k,1)$
3: $\quad\quad$ **for** $l = 1 \rightarrow k$ **do**
3: $\quad\quad\quad \mathcal{I}[l] \leftarrow p$
3: $\quad\quad\quad$ **table** $[l] \leftarrow f(\mathcal{H},\mathcal{I})$
3: $\quad\quad\quad \mathcal{I} \leftarrow \mathcal{S}$
4: $\quad\quad$ **end for**
5: $\quad\quad$ **if** $\min$ (**table**) $< \mathrm{MSE}^*$ **then**
5: $\quad\quad\quad \mathrm{MSE}^* \leftarrow \min$ (**table**)
5: $\quad\quad\quad \mathcal{S}[\arg\min$ (**table**)$] \leftarrow p$
6: $\quad\quad$ **end if**
7: $\quad$ **end while**
8: **end for**

Algorithm 1: *Greedy Approach for Antenna Selection*

2.2 Blind Antenna Selection

Assumption 1 *We assume that n, m and k grow simultaneously large while*

1. $\frac{n}{m} \to c \in (1,\infty)$

2. $0 < \liminf \frac{k}{n} < \limsup \frac{k}{n} < 1.$

3. $\liminf \frac{k}{m} > 1.$

Assumption 2 *The correlation matrix $\mathbf{R}$ satisfies*

1. $\sup_n \|\mathbf{R}\| < \infty$

2. $\inf_n \frac{1}{n}\mathrm{tr}\,\mathbf{R} > 0.$

Lemma 1 [1] *Let Assumptions 1 and 2 hold true. Let δ be the unique solution to the following equation*

$$\delta = m\left(\mathrm{tr}\left[\mathbf{R}\mathrm{diag}(s)\left(\mathbf{I}_n + \delta\mathbf{R}\mathrm{diag}(\mathbf{s})\right)^{-1}\right]\right)^{-1} \tag{8}$$

Define $\overline{\mathrm{MSE}}(\mathbf{s})$ as

$$\overline{\mathrm{MSE}}(\mathbf{s}) = \delta$$

Then, $\mathrm{MSE}(\mathbf{s})$ satisfies

$$\mathrm{MSE}(\mathbf{s}) - \overline{\mathrm{MSE}}(\mathbf{s}) \xrightarrow[n\to\infty]{a.s.} 0.$$

$\overline{\mathrm{MSE}}(\mathbf{s})$ only depends on the channel statistics $\mathbf{R}$.

Theorem 1 *1. The function*

$$\begin{aligned}\overline{\mathrm{MSE}} : \mathbb{R}_+^n &\to \mathbb{R}_+ \\ \mathbf{s} &\mapsto \overline{\mathrm{MSE}}(\mathbf{s})\end{aligned}$$

is convex in $\mathbb{R}_+^n$.

2.

$$\frac{\partial\overline{\mathrm{MSE}}(\mathbf{s})}{\partial s_i} = \frac{\delta\left[\mathbf{R}^{\frac{1}{2}}\left(\mathbf{I} + \delta\mathbf{R}\,\mathrm{diag}(\mathbf{s})\right)^{-2}\mathbf{R}^{\frac{1}{2}}\right]_{i,i}}{\mathrm{tr}\left[\mathbf{R}\,\mathrm{diag}(\mathbf{s})\left(\mathbf{I} + \delta\mathbf{R}\,\mathrm{diag}(\mathbf{s})\right)^{-2}\right]}. \tag{9}$$

Corollary 1 *Convex optimization techniques can be applied to solve problems with an objective given by $\overline{\mathrm{MSE}}(\mathbf{s})$.*

As a matter of fact, the blind antenna selection problem can be formulated as follows

$$\begin{aligned}\hat{\mathbf{s}}_{blind} = &\underset{\mathbf{s}}{\arg\min} \quad \overline{\mathrm{MSE}}(\mathbf{s}) \\ \text{s.t.} \qquad &\mathbf{1}^T\mathbf{s} = k \\ &s_i \in \{0,1\},\ i = 1,\cdots,n.\end{aligned} \tag{10}$$

Similarly, (10) can be solved using both heuristics: the convex relaxation and the greedy approach.

3. Numerical Results and Discussion

$m = 30$ users, $n = 100$ antennas and $\rho = 20$ dB.

We consider the correlation model given by

$$\mathbf{R}_{i,j} = \exp\left[-0.05 d^2 (i-j)^2\right],\ 1 \le i,j \le n. \tag{11}$$

3.1 Complexity

Algorithm	Complexity
Convex Optimization(Channel-aware)	$N \times \mathcal{O}(n^3)$
Convex Optimization(Blind)	$\mathcal{O}(n^3)$
Greedy(Channel-aware)	$K \times N \times \mathcal{O}(n^2)$
Greedy(Blind)	$K \times \mathcal{O}(n^2)$

Table 1: *Computational complexity of the different proposed algorithms.*

3.2 MSE Performance

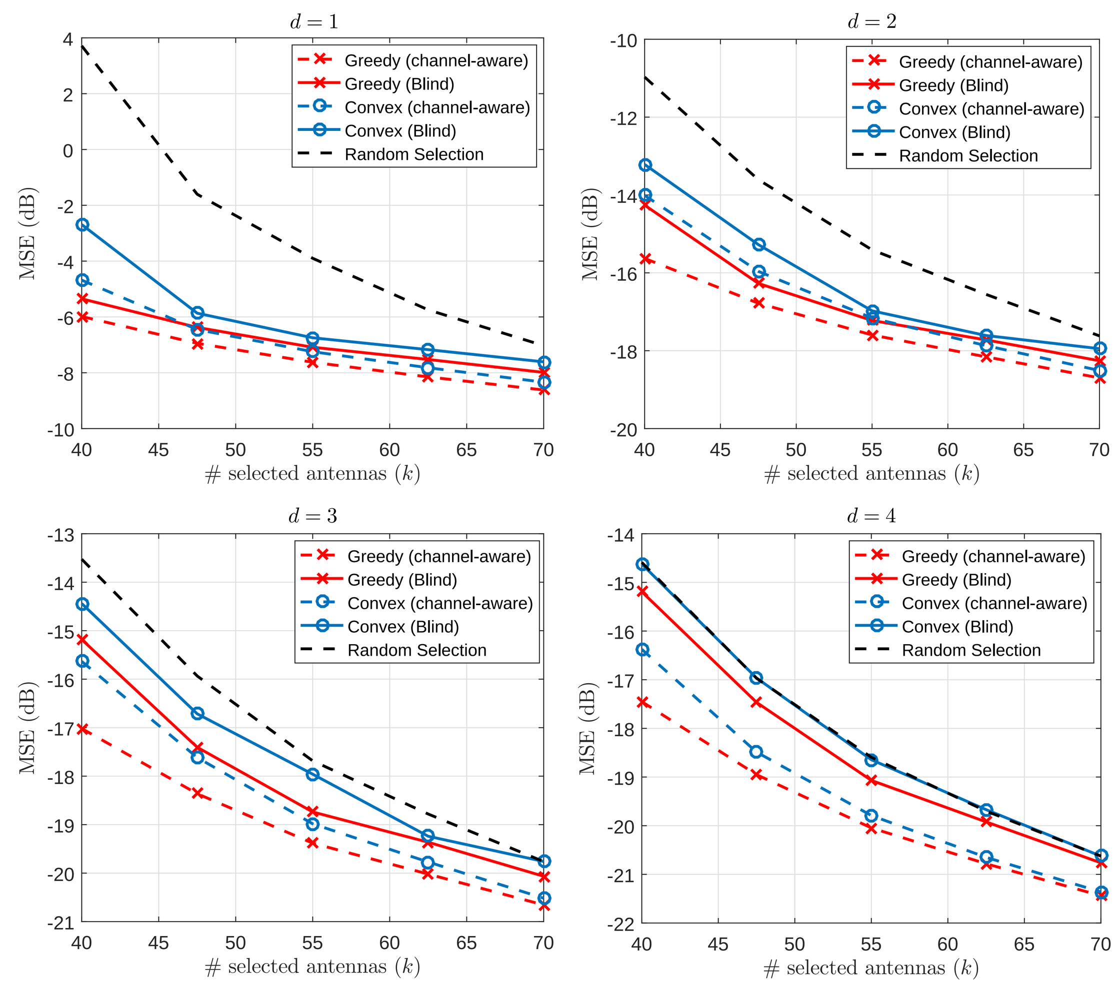

Figure 2: *Average MSE achieved by the proposed selection techniques versus k for different values of the antennas' separation d.*

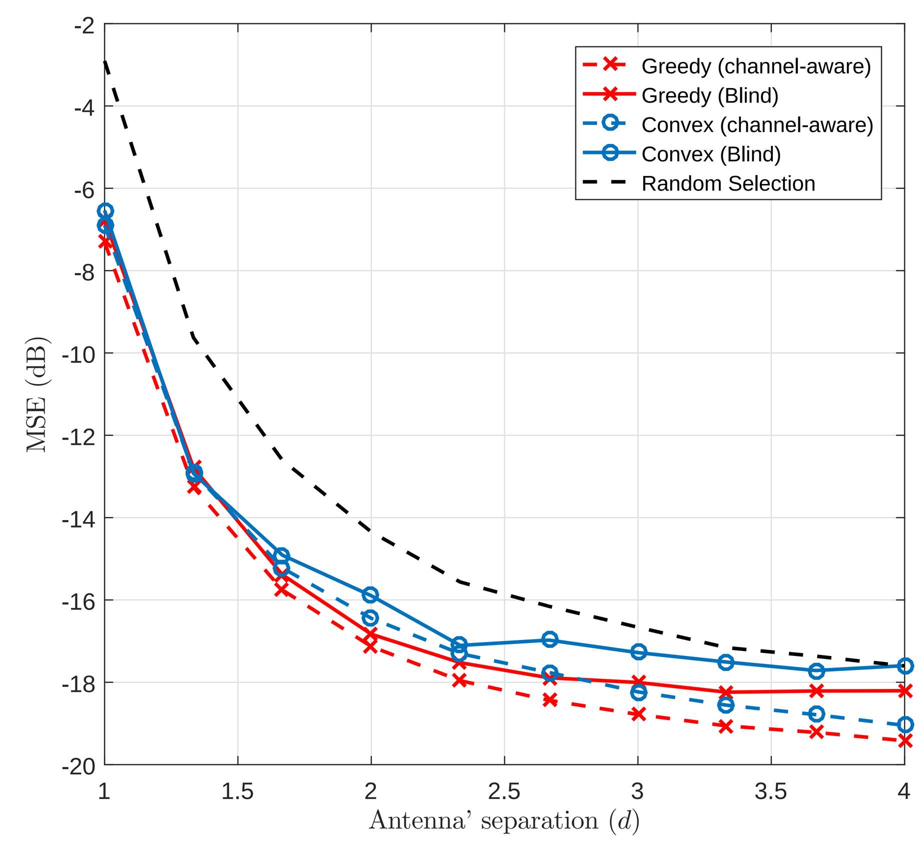

Figure 3: *Average MSE achieved by the proposed selection techniques versus the antennas' separation d with $k = 50$.*

Comments

- When the correlation between antennas is low ($d = 4$), the proposed blind algorithms are not that advantageous as compared to the random selection algorithm.
- However, with the impact of correlation becoming more important ($d \downarrow$), the gain of blind approaches over the random selection approach increases.
- Blind algorithms perform antenna selection at the pace of the variation of the large scale statistics. A high reduction in the computational complexity is thus achieved compared to channel-aware algorithms.
- From a practical point of view, blind selection algorithms are more suitable since they consider practical issues such as antenna synchronization and adaptation.

4. Conclusion

In this work, we showed that using tools from random matrix theory, it is possible to asymptotically approximate the MSE. As such, perfect knowledge of the channel matrix is not needed and only statistics are required to perform selection. We proposed two techniques: the first is based on a greedy approach and the second is based on a convex relaxation heuristic. Numerical results showed that the blind techniques have a comparable performance to techniques that require full knowledge of the channel matrix, especially at high correlation.

5. References

[1] J. W. Silverstein and Z. D. Bai, "On the Empirical Distribution of Eigenvalues of a Class of Large Dimensional Random Matrices," *Journal of Multivariate Analysis*, vol. 54, pp. 175–192, May 2002.